

NONDEGENERACY OF CRITICAL POINTS OF THE MEAN CURVATURE OF THE BOUNDARY FOR RIEMANNIAN MANIFOLDS

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ABSTRACT. Let M be a compact smooth Riemannian manifold of finite dimension $n + 1$ with boundary ∂M and ∂M is a compact n -dimensional submanifold of M . We show that for generic Riemannian metric g , all the critical points of the mean curvature of ∂M are nondegenerate.

Let M be a connected compact C^m manifold with $m \geq 3$ of finite dimension $n + 1$ with boundary ∂M . The boundary ∂M is a compact C^m n -dimensional submanifold of M . Let \mathcal{M}^m be the set of all C^m Riemannian metrics on M . Given a metric $\bar{g} \in \mathcal{M}^m$, we consider the mean curvature of the boundary ∂M of (M, \bar{g}) . Our goal is to prove that generically for a Riemannian metric g all the critical points of the mean curvature of the boundary ∂M of (M, g) are nondegenerate. More precisely we show the following result

Theorem 1. *The set*

$$(1) \quad \mathcal{A} = \left\{ g \in \mathcal{M}^m : \begin{array}{l} \text{all the critical points of the} \\ \text{mean curvature of the boundary} \\ \text{of } (M, g) \text{ are nondegenerate} \end{array} \right\}$$

is an open dense subset of \mathcal{M}^m .

We denote by \mathcal{S}^m the space of all C^m symmetric covariant 2-tensors on M and \mathcal{B}_ρ the ball in \mathcal{S}^m of radius ρ . The set \mathcal{M}^m of all C^m Riemannian metrics on M is an open subset of \mathcal{S}^m .

A possible application of this result arises in the study of the following Neumann problem

$$(2) \quad \begin{cases} -\varepsilon \Delta_g u + u = u^{p-1} & \text{in } M \\ u > 0 & \text{in } M \\ \frac{\partial u}{\partial \nu} = 0 & \text{on } \partial M \end{cases}$$

where $p > 2$ if $n = 2$ and $2 < p < 2^* = \frac{2n}{n-2}$ if $n \geq 3$, ν is the external normal to ∂M and ε is a positive parameter. In [1] the authors prove that the problem (2) has a mountain pass solution u_ε which has a unique maximum point $\xi_\varepsilon \in \partial M$ converging, as $\varepsilon \rightarrow 0$, to a maximum point of the mean curvature of the boundary. Recently, in [4] a relation between topology of the boundary ∂M and the the number of solutions is established. More precisely it has been proved that Problem (2) has at

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least cat ∂M non trivial solution provided ε small enough. In a forthcoming paper [3] the author shows that nondegenerate critical points of the mean curvature of the boundary produce one peak solutions of problem (2).

In our opinion the role of the mean curvature of the boundary in Problem (2) on a manifold M with boundary is similar to the role of the scalar curvature in the problem

$$\begin{cases} -\varepsilon \Delta_g u + u = u^{p-1} & \text{in } M \\ u > 0 & \text{in } M \end{cases}$$

defined on a boundaryless manifold M . Recently in some papers [2, 5, 6] assuming a sort of non degeneracy of the critical points of the scalar curvature of the boundaryless manifold (M, g) some results of existence of one peak and multipeak solutions have been proved. Moreover, in [7] it is proved that generically with respect to the metric g all the critical points of the scalar curvature are nondegenerate, so the latter result of non degeneracy can be applied to the previous existence theorems.

1. SETTING OF THE PROBLEM

In the following, with abuse of notation we often identify a point in the manifold with its Fermi coordinates (x_1, \dots, x_n, t) . We now recall the definition of Fermi coordinates.

Let $\bar{\xi}$ belongs to the boundary ∂M , let (x_1, \dots, x_n) be coordinates on the n manifold ∂M in a neighborhood of a point $\bar{\xi}$. Let $\gamma(t)$ be the geodesic leaving from $\bar{\xi}$ in the orthogonal direction to ∂M and parametrized by arc lenght. Then the set (x_1, \dots, x_n, t) are the so called *Fermi coordinates* at $\bar{\xi} \in \partial M$ where $(x_1, \dots, x_n) \in B_{\bar{g}}(0, R)$ and $0 \leq t < T$ for R, T small enough.

In these coordinates the arclenght ds^2 is written as

$$ds^2 = dt^2 + g_{ij}(x, t) dx_i dx_j \text{ for } i, j = 1, \dots, n.$$

Also, we set $I(\bar{\xi}, R)$ the neighborhood of $\bar{\xi}$ such that, in Fermi coordinates, $|x| = |(x_1, \dots, x_n)| < R$ and $0 \leq t < R$.

If \bar{g} is the metric of the manifold M then $\det \bar{g} = \det (g_{ij})_{ij}$.

We denote by $h_{ij}(x, t)$ the second fundamental form of the submanifold

$$\partial M_t = \{(x, t) : x \in \partial M, 0 \leq t < T\}$$

for T small enough. Moreover, $H^{\bar{g}}(x, t)$ is the trace of the second fundamental form $h_{ij}(x, t)$ of the submanifold ∂M_t , that is

$$(3) \quad H^{\bar{g}}(x, t) = (\bar{g}(x, t))^{ij} h_{ij}(x, t)$$

By a well known result of Escobar [11] we have that the second fundamental form in a neighborhood of a point $\bar{\xi} \in \partial M$ can be expressed in term of the metric \bar{g} of the manifold in the following way

$$(4) \quad \partial_t \bar{g}_{ij}(x, t) = -2h_{ij}(x, t).$$

where (x, t) are the Fermi coordinates centered at $\bar{\xi}$.

We denote by \mathcal{S}^m the Banach space of all C^m symmetric covariant symmetric 2-tensors on M . The norm $\|\cdot\|_m$ is defined in the following way. We fix a finite covering $\{V_\alpha\}_{\alpha \in L}$ of M such that the closure of V_α is contained in U_α where $\{U_\alpha, \psi_\alpha\}$ is an open coordinate neighborhood. Let $V_\alpha \cap \partial M = \emptyset$. If $k \in \mathcal{S}^m$ we denote by

k_{ij} the components of k with respect to the coordinates (x_1, \dots, x_{n+1}) on V_α . We define

$$\|k\|_m = \sum_{\alpha \in L} \sum_{|\beta| \leq m} \sum_{i,j=1}^{n+1} \sup_{\psi_\alpha(V_\alpha)} \frac{\partial^\beta k_{i,j}}{\partial x_1^{\beta_1} \cdots \partial x_m^{\beta_m}}$$

We proceed in analogous way when $V_\alpha \cap \partial M \neq \emptyset$, by means of Fermi coordinates.

We recall and abstract result in transversality theory (see [8, 9, 10]) which will be fundamental for our results.

Theorem 2. *Let X, Y, Z be three real Banach spaces and let $U \subset X$, $V \subset Y$ be two open subsets. Let F be a C^1 map from $V \times U$ in to Z such that*

- (1) *For any $y \in V$, $F(y, \cdot) : x \rightarrow F(y, x)$ is a Fredholm map of index 0.*
- (2) *0 is a regular value of F , that is $F'(y_0, x_0) : Y \times X \rightarrow Z$ is onto at any point (y_0, x_0) such that $F(y_0, x_0) = 0$.*
- (3) *The map $\pi \circ i : F^{-1}(0) \rightarrow Y$ is σ -proper, that is $F^{-1}(0) = \bigcup_{s=1}^{\infty} C_s$ where C_s is a closed set and the restriction $\pi \circ i|_{C_s}$ is proper for any s . Here $i : F^{-1}(0) \rightarrow Y \times X$ is the canonical embedding and $\pi : Y \times X \rightarrow Y$ is the projection.*

then the set $\theta = \{y \in V : 0 \text{ is a regular value of } F(y, \cdot)\}$ is a residual subset of V , that is $V \setminus \theta$ is a countable union of closed subsets without interior points.

At this point we introduce the C^1 map

$$(5) \quad \begin{aligned} F & : \mathcal{B}_\rho \times B(0, R) \rightarrow \mathbb{R}^n \\ F(k, x) & = \nabla_x H^{\bar{g}+k}(x, t) \Big|_{(x,0)} \end{aligned}$$

where $H^{\bar{g}+k}(x, t)$ is the mean curvature of ∂M_t related to the metric $\bar{g} + k$ at the point (x, t) . This map is C^1 if $m \geq 3$. Moreover, by (3) and (4) we have

$$(6) \quad F(k, x) = -\frac{1}{2} \nabla_x \left((\bar{g} + k)^{ij}(x, t) \partial_t (\bar{g} + k)_{ij}(x, t) \right) \Big|_{(x,0)}$$

Lemma 3. *The set*

$$\mathcal{A} = \left\{ g \in \mathcal{M}^m : \begin{array}{l} \text{all the critical points of the} \\ \text{mean curvature of the boundary} \\ \text{of } (M, g) \text{ are nondegenerate} \end{array} \right\}$$

is an open set in \mathcal{M}^m .

Proof. If $\bar{g} \in \mathcal{A}$, we have that the critical point of the mean curvature of ∂M are in a finite number, say ξ_1, \dots, ξ_ν . We consider the Fermi coordinates in a neighborhood of ξ_1 , and the map F defined in (5) and (6).

We have that $F(0, 0) = 0$ and that $\partial_x F(0, 0) : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is an isomorphism because ξ_1 is a nondegenerate critical point. Thus by the implicit function theorem there exist two positive numbers ρ_1 and R_1 and a unique function $x_1(k)$ such that in $\mathcal{B}_{\rho_1} \times B(0, R_1)$ the level set $\{F(k, x) = 0\}$ is the graphic of the function $\{x = x_1(k)\}$.

We can argue analogously for ξ_2, \dots, ξ_ν , finding constant $\rho_2, R_2, \dots, \rho_\nu, R_\nu$ for which the set $\{F(k, x) = 0\}$ in a neighborhood of $\mathcal{B}_{\rho_2} \times B(0, R_2), \dots, \mathcal{B}_{\rho_\nu} \times B(0, R_\nu)$ can be respectively described by means of the functions $x_2(k), \dots, x_\nu(k)$.

We set $B_i = \{\xi \in \partial M : d_g(\xi, \xi_i) < R_i\}$. We claim that there are no critical points of the mean curvature for the metric $\bar{g} + k$ in the set $\partial M \setminus \bigcup_{i=1}^\nu B_i$ for any $k \in \mathcal{B}_\rho$, provided ρ sufficiently small. Otherwise we can find a sequence of $\{\rho_n\}_n \rightarrow 0$, a sequence $k_n \in \mathcal{B}_{\rho_n}$ and a sequence of points $\xi_n \in \partial M \setminus \bigcup_{i=1}^\nu B_i$ such

that $F(k_n, \xi_n) = 0$. But, by compactness of ∂M , $\xi_n \rightarrow \bar{\xi}$ for some $\bar{\xi} \in \partial M \setminus \cup_{i=1}^{\nu} B_i$ and, by continuity of F , $\bar{\xi}$ is such that $F(0, \bar{\xi}) = 0$, that is a contradiction.

At this point the proof is complete. \square

2. PROOF OF THE MAIN THEOREM

We are going to apply the transversality Theorem 2 to the map F defined in (5). In this case we have $X = Z = \mathbb{R}^n$, $Y = \mathcal{S}^m$, $U = B(0, R)$ and $V = \mathcal{B}_\rho$ with R and ρ small enough. Since X and Z are finite dimensional, it is easy to check that for any $k \in \mathcal{B}_\rho$ the map $x \mapsto F(k, x)$ is Fredholm of index 0, so assumption (i) holds.

To prove assumption (ii) we will show, in Lemma 5, that, if the pair $(\tilde{x}, \tilde{k}) \in B(0, R) \times \mathcal{B}_\rho$ is such that $F(\tilde{k}, \tilde{x}) = 0$, the map $F'_k(\tilde{k}, \tilde{x})$ defined by

$$k \rightarrow D_k \nabla_x H^{\bar{g}+\tilde{k}}(x, t) \Big|_{(\tilde{x}, 0)} [k]$$

is surjective.

As far as it concerns assumption (iii) we have that

$$F^{-1}(0) = \cup_{s=1}^{\infty} C_s \text{ where } C_s = \left\{ \overline{B(0, R - 1/s)} \times \overline{\mathcal{B}_{\rho-1/s}} \right\} \cap F^{-1}(0).$$

It is easy to check that the restriction $\pi \circ i|_{C_s}$ is proper, that is if the sequence $\{k_n\}_n \subset \overline{\mathcal{B}_{\rho-1/s}}$ converges to k_0 in \mathcal{S}^m and the sequence $\{x_n\}_n \subset \overline{B(0, R - 1/s)}$ is such that $F(k_n, x_n) = 0$ then by compactness of $\overline{B(0, R - 1/s)}$ there exists a subsequence of $\{x_n\}_n$ converging to some $x_0 \in \overline{B(0, R - 1/s)}$ and $F(k_0, x_0) = 0$.

So we are in position to apply Theorem 2 and we get that the set

$$\begin{aligned} A(\bar{\xi}, \rho) &= \left\{ \begin{array}{l} k \in \mathcal{B}_\rho : F'_k(x, k) : \mathbb{R}^n \rightarrow \mathbb{R}^n \text{ is onto} \\ \text{at any point } (x, k) \text{ s.t. } F(x, k) = 0 \end{array} \right\} \\ &= \left\{ \begin{array}{l} k \in \mathcal{B}_\rho : F'_k(x, k) : \mathbb{R}^n \rightarrow \mathbb{R}^n \text{ is invertible} \\ \text{at any point } (x, k) \text{ s.t. } F(x, k) = 0 \end{array} \right\} \\ &= \left\{ \begin{array}{l} k \in \mathcal{B}_\rho : \text{the critical points } \xi \in I(\bar{\xi}, R) \subset \partial M \\ \text{of the mean curvature of } \partial M \text{ are non degenerate} \end{array} \right\} \end{aligned}$$

is a residual subset of \mathcal{B}_ρ . Since M is compact, there exists a finite covering $\{I(\xi_i, R)\}_{i=1, \dots, l}$ of ∂M , where $\xi_1, \dots, \xi_l \in \partial M$. For any index i there exists a residual set $A(\xi_i, \rho) \subset \mathcal{B}_\rho$ such that the critical points of the curvature in $I(\xi_i, R)$ are non degenerate for any $k \in A(\xi_i, \rho)$. Let

$$A(\rho) = \cap_{i=1}^l A(\xi_i, \rho).$$

Then the set $A(\rho)$ is a residual set in \mathcal{B}_ρ .

We now may conclude that, given the metric \bar{g} , for any ρ small enough there exists a $\bar{k} \in A(\rho) \subset \mathcal{B}_\rho$ such that the critical points of the mean curvature of ∂M related to the metric $\bar{g} + \bar{k}$ are non degenerate. Thus the set \mathcal{A} defined in (1) is a dense set. Moreover, by Lemma 3 we have that \mathcal{A} is open and the proof of Theorem 1 is complete.

3. TECHNICAL LEMMAS

We now prove two technical lemmas in order to obtain assumption (ii) of the transversality theorem.

Lemma 4. *For any $x \in B(0, R) \subset \mathbb{R}^n$ and for any $\tilde{k} \in \mathcal{B}_\rho \subset \mathcal{S}^m$ it holds*

$$\begin{aligned} F'_k(\tilde{k}, x)[k] &= (\tilde{g}^{-1}k\tilde{g}^{-1})_{im}\tilde{g}_{ml,s}\tilde{g}^{li}\tilde{g}_{ij,t}|_{x,0} + \tilde{g}^{im}k_{ml,s}\tilde{g}^{lj}\tilde{g}_{ij,t}|_{x,0} + \tilde{g}^{im}\tilde{g}_{ml,s}(\tilde{g}^{-1}k\tilde{g}^{-1})_{lj}\tilde{g}_{ij,t}|_{x,0} \\ &\quad (7) - \tilde{g}^{im}\tilde{g}_{ml,s}\tilde{g}^{lj}k_{ij,t}|_{x,0} - (\tilde{g}^{-1}k\tilde{g}^{-1})_{ij}\tilde{g}_{ij,ts}|_{x,0} + \tilde{g}^{ij}k_{ij,ts}|_{x,0} \end{aligned}$$

Proof. We differentiate the identity $g^{im}g_{mj} = \delta_{ij}$, obtaining

$$\partial_{x_s} g^{ij} := g^{ij}_{,s} = -g^{im}g_{ml,s}g^{lj}$$

Then we have

$$\begin{aligned} (8) \quad \partial_{x_s} \left[(g+k)^{ij}(x,t)(g+k)_{ij,t}(x,t) \right] &= \\ &= -(g+k)^{im}(g+k)_{ml,s}(g+k)^{lj}(g+k)_{ij,t} + (g+k)^{ij}(g+k)_{ij,ts}. \end{aligned}$$

Here $\partial_t g_{ij} := g_{ij,t}$. We recall that, if ρ is sufficiently small, for any pair $k, \tilde{k} \in \mathcal{B}_\rho$, we have

$$(9) \quad (g + \tilde{k} + k)^{-1} = (\tilde{g} + k) = \tilde{g}^{-1} - \tilde{g}^{-1}k\tilde{g}^{-1} + \sum_{\lambda=2}^{\infty} (-1)^\lambda (\tilde{g}^{-1}k)^\lambda \tilde{g}^{-1}.$$

Here $\tilde{g} = g + \tilde{k}$. At this point, by (8) and by (9), we have

$$\begin{aligned} D_k \partial_{x_s} \left((g+k)^{ij}(x,t)(g+k)_{ij,t}(x,t) \right) \Big|_{\tilde{k},x,0} [k] &= \\ &= D_{\tilde{g}} \left((\tilde{g})^{ij}(x,t)(\tilde{g})_{ij,t}(x,t) \right) \Big|_{\tilde{g},x,0} [k] = \\ &= (\tilde{g}^{-1}k\tilde{g}^{-1})_{im}\tilde{g}_{ml,s}\tilde{g}^{li}\tilde{g}_{ij,t}|_{x,0} + \tilde{g}^{im}k_{ml,s}\tilde{g}^{lj}\tilde{g}_{ij,t}|_{x,0} + \tilde{g}^{im}\tilde{g}_{ml,s}(\tilde{g}^{-1}k\tilde{g}^{-1})_{lj}\tilde{g}_{ij,t}|_{x,0} \\ &\quad - \tilde{g}^{im}\tilde{g}_{ml,s}\tilde{g}^{lj}k_{ij,t}|_{x,0} - (\tilde{g}^{-1}k\tilde{g}^{-1})_{ij}\tilde{g}_{ij,ts}|_{x,0} + \tilde{g}^{ij}k_{ij,ts}|_{x,0}. \end{aligned}$$

This concludes the proof \square

Lemma 5. *For any (\tilde{x}, \tilde{k}) such that $F(\tilde{k}, \tilde{x}) = 0$ we have that the map*

$$(x, k) \mapsto F'_k(\tilde{k}, \tilde{x})[k] + F'_x(\tilde{k}, \tilde{x})x$$

is onto on \mathbb{R}^n .

Proof. Let (\tilde{x}, \tilde{k}) such that $F(\tilde{k}, \tilde{x}) = 0$. To obtain our claim it is sufficient to prove that the map $F'_k(\tilde{k}, \tilde{x})[k] : \mathcal{S}^m \rightarrow \mathbb{R}^n$ is onto. More precisely, we are going to prove that, given e_1, \dots, e_n the canonical base in \mathbb{R}^n , for any $\nu = 1, \dots, n$ there exists $k \in \mathcal{S}^m$ such that $F'_k(\tilde{k}, \tilde{x})[k] = e_\nu$. We remark that the ontoness is independent from the choice of the coordinates, so, for any $\tilde{\xi} = (\tilde{x}, 0) \in \partial M$ we choose the exponential coordinates in ∂M with metric $g + \tilde{k}$ centered in $\tilde{\xi}$, so we have to prove simply that, given ν , there exists $k \in \mathcal{S}^m$ such that

$$\begin{aligned} D_k \partial_{x_\nu} \left((g+k)^{ij}(x,t)(g+k)_{ij,t}(x,t) \right) \Big|_{\tilde{k},x,0} [k] &= 1 \\ D_k \partial_{x_s} \left((g+k)^{ij}(x,t)(g+k)_{ij,t}(x,t) \right) \Big|_{\tilde{k},x,0} [k] &= 0 \end{aligned}$$

for all $s \neq \nu$.

We use (7) of Lemma 4. Using Fermi coordinates we have that the metric $\tilde{g}(\cdot, 0)$ on the submanifold ∂M has the form $\tilde{g}_{ij}(0, 0) = \delta_{ij}$, thus

$$(10) \quad D_k \partial_{x_s} \left((g+k)^{ij}(x, t) (g+k)_{ij,t}(x, t) \right) \Big|_{\tilde{k}, 0, 0} [k] = \\ k_{im} \tilde{g}_{mj,s} \tilde{g}_{ij,t} \Big|_{0,0} - k_{ij,s} \tilde{g}_{ij,t} \Big|_{0,0} + \tilde{g}_{il,s} k_{lj} \tilde{g}_{ij,t} \Big|_{0,0} \\ - \tilde{g}_{ij,s} k_{ij,t} \Big|_{0,0} - k_{ij} \tilde{g}_{ij,ts} \Big|_{0,0} + k_{ii,ts} \Big|_{0,0}$$

Now, we choose $k \in \mathcal{S}^m$ such that the map $x \mapsto k_{ij}(x, 0)$ vanishes at $x = 0$ for all i, j . Then by (10) we have

$$D_k \partial_{x_s} \left((g+k)^{ij}(x, t) (g+k)_{ij,t}(x, t) \right) \Big|_{\tilde{k}, 0, 0} [k] = \\ - k_{ij,s} \tilde{g}_{ij,t} \Big|_{0,0} - \tilde{g}_{ij,s} k_{ij,t} \Big|_{0,0} + k_{ii,ts} \Big|_{0,0}$$

Moreover we assume the the first derivatives $k_{ij,s}(0, 0) = k_{ij,t}(0, 0) = 0$ for all i, j . Then

$$(11) \quad D_k \partial_{x_s} \left((g+k)^{ij}(x, t) (g+k)_{ij,t}(x, t) \right) \Big|_{\tilde{k}, 0, 0} [k] = k_{ii,ts} \Big|_{0,0}.$$

We now prove the claim for $\nu = 1$. Let us choose $k \in \mathcal{S}^m$ such that

$$k_{11}(x, t) = x_1 t; \quad k_{ij}(x, t) = 0$$

for $(i, j) \neq (1, 1)$. Thus (11) rewrites as

$$D_k \partial_{x_1} \left((g+k)^{ij}(x, t) (g+k)_{ij,t}(x, t) \right) \Big|_{\tilde{k}, 0, 0} [k] = k_{11,ts} \Big|_{0,0} = 1 \\ D_k \partial_{x_s} \left((g+k)^{ij}(x, t) (g+k)_{ij,t}(x, t) \right) \Big|_{\tilde{k}, 0, 0} [k] = k_{ss,ts} \Big|_{0,0} = 0$$

for $s = 2, \dots, n$. Analogously we proceed for any $\nu = 2, \dots, n$ and we conclude the proof. \square

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